Identifying the share of internal and external constraints of power plant units from the power allocated in the production setup

Abstract

Planning the power generation units is the main aim for operators of the Iranian electricity industry to access an applicable production setup for the grid units with the minimum possible cost and take into account the security constraints of the system. Whereas in most cases, internal and external limitations prevent accepting power from more economical power plant units. Identifying the impact of constraints is a possibility for the producers. This is based on the comparison of four different mechanisms in the current method used in Iran's electricity market. In this study, while introducing the used method in Iran's electricity market, an optimization-based model was provided to identify the share of internal and external constraints in the accepted power of power plants. The proposed method was implemented with high accuracy. The model was implemented on a 3-bus sample power system and an IEE 118-bus system. The findings showed the efficiency and accuracy of the proposed method.

Keywords: Daily production setup of power plant units; Constraints of power plant; Settlement of the electricity market; Decomposed binominal coefficient

Introduction

how power plant units participate in the electricity market is one of the considerable approvals of the Electricity Market Regulatory Board since the beginning of Iran's electricity market. This is a mathematical optimization problem with the objective function of reducing operating costs and is bound by security and power plant constraints, which its results are affected by non-convexity. While there is a long research background in markets under convexity assumptions, non-convexity exists in most electricity markets. Non-convexity is mainly due to several reasons, including minimum production, start-up or power cut costs, and inseparable and unavoidable costs [1]. Therefore, the dual variables in the offers and limitations of exploitation make the electricity market auction non-convex. In this situation, the marginal price of the system cannot guarantee a return on the total cost of production [2]. In some economic theories, the issue of "pricing" has been discussed in markets with non-convexity, and most of them follow the reference of [3] and the main results of the convex economic theory. Whereas, pricing in non-convexity markets has attracted the attention of researchers after the emergence of the electricity market in the United States and around the world [2]. The proposed methods for electricity market pricing can be generally categorized according to Table 1.

Table 1) Classification of suggested methods for pricing in non-convexity markets

Reference	Main characteristic	Proposed method
[4]	Turning mixed integer linear programming (MILP) to linear programming (LP) using fixed integer variables.	Integer programming (IP)
[5, 6]	In addition to integer variables, some continuous variables are fixed in optimal values.	Modified integer programming
[7, 8]	The non-convexity costs in the MILP are linearized using a convex hull so that uplift payment will be minimized.	Minimum Uplift Convex Hull
[9, 10]	The convex hull linearizes the non-convex prices, so that the sum of additional payments is zero.	General additional payments
[2]	Prices higher than marginal costs are selected, so that those unfair payments received by the beneficiary units are allocated to the affected units.	Additional minimum payments
[11]	The prices are selected based on the maximum average cost of the players.	Average cost
[12]	Some equality constraints that have an integer variable are released.	Semi-Lagrange Relaxation
[13, 14, 18]	Fixing the dual variables and deriving the duality problem and establishing the strong dual theorem	Primal-Dual
[15, 16]	Fixing the dual variables and extracting the binary problem and its solving and extracting the marginal prices	Dual pricing

Some of these plans have been implemented in real markets and have affected the efficiency and effectiveness of electricity markets. The methods implemented in the markets can be divided into two categories [17]. The first approach is American (such as PJM, NYISO, ISO-NE, MISO, ERCOT, and New Zealand), which considers the solution of maximum social welfare, although they have deviated from the linear pricing. The second approach is mostly found in Europe (such as NordPool, CWE, etc.), which implements correct linear pricing, although pricing is achieved through a sub-optimal solution in terms of social welfare. In the current structure of Iran's electricity market, a mechanism is defined based on the comparison of the results of four main models (Table 2).

Table 2) Four used mechanisms in Iran's electricity market

mechanism	constraints
Economic arrangement (ECO)	Objective function: Minimizing the purchase cost from the power plant and the fuel difference; Constraints: network repairs and exchanges.
A technical and economic arrangement (**REQ**)	Objective function: Minimizing the cost of purchasing from the power plant and fuel differences and the cost of denied opportunity; Constraints: Network repairs, exchanges, and restrictions related to all power plants.
Economic arrangement considering the limitations of PP power plant	Objective function: Minimizing the cost of purchasing fuel from the power plant and Fuel difference

(ECO+pp)	Constraints: Network repairs, exchanges, and constraints related to the PP power plant.
Technical and economic arrangement without taking into account the limitations of PP power plant (REQ-pp)	Objective function: Network repairs and exchanges and restrictions related to all power plants except the PP power plant; Constraints: Network repairs, exchanges, and restrictions related to all power plants except the PP power plant.

In the current method, to calculate the payment to the power plant units, the market production arrangement for four different modes is determined through the optimization problem. Thus, the optimization is implemented in the arrangement of REQ-PP and ECO+pp according to the number of power plants. This trend causes the time-consuming market settlement process and the concern of players about the accuracy of calculating the internal limitation of power plant units in the accepted power in other words, it has become a unit limitation (UL) and an opportunity cancellation (OC). Also, since in the current method, the effect of internal and external restrictions for each power plant is examined simultaneously, it is not possible to identify the contribution of each of the constraints on the power assigned to the power plant. For this reason, the current method is less transparent to distinguish between different power plant units with different technologies in power generation. For example, the approach for flexible players such as gas power plants, which have the technical advantage of decreasing and increasing production in the time horizon of their operation, is the same with actors who have little flexibility. According to the mentioned explanations, the purpose of this article is to present a model to identify and calculate the sensitivity of power allocated to power plant units in the production arrangement concerning the internal and external constraints of power plant units. The prominent feature of the proposed model is being able to identify the mentioned sensitivity level without the need to implement multiple arrangements and only through the output of the technical-economic arrangement (REC). Also, the model calculates the opportunity to produce deprived power of the power plant units, and its included in the bills of the actors. By identifying the share of constraints on the effectiveness of each power plant unit in the objective function, the electricity market becomes closer to a competitive market and the income of energy selling will be distributed more fairly among the players. In the second section of this article, the mathematical formulation of the desired method is provided to identify the contribution of internal and external constraints of power plant units in the accepted power. In the third section, the issue of market settlement is stated. The numerical results obtained from the implementation of the proposed model are reviewed and evaluated in the fourth section. The results of this study are provided in section 5.

Proposed method in identifying the share of constraints

The production arrangement of power plant units can be summarized as a MILP optimization problem with the objective function of market exploitation costs along with the constraints of the network and power plant units by relations 1-3. In these relationships, x and u represent the vector of continuous and discrete variables of the model.

$$\text{minimize} \quad \mathbf{c}^{\mathbf{T}}\mathbf{x} \tag{1}$$

$$\mathbf{a}^{T}\mathbf{x}+\mathbf{h}^{T}\mathbf{u}=\mathbf{b} \ :\boldsymbol{\lambda} \tag{2}$$

$$\mathbf{e}^{T}\mathbf{x}+\mathbf{f}^{T}\mathbf{u}\geq\mathbf{d} \tag{3}$$

By fixing the discrete variables in the optimal values ($\mathbf{u}^{*}$) obtained from the model (1)-(3), the MILP problem becomes an LP and convex problem (the details are provided in Appendix 1), in which the market balance is established and optimal conditions are established for the *i*-th participant. Therefore, the problem can be described as relations (4)-(6):

$$\text{minimize} \quad \mathbf{c}^{\mathbf{T}}\mathbf{x} \tag{4}$$

$$\mathbf{a}_{\mathbf{i}}^{\mathbf{T}}\mathbf{x}=b_i' : \lambda_i \quad \forall i=1,\ldots,m \tag{5}$$

$$\mathbf{e}_j^{T}\mathbf{x}\geq d_j' \quad : \mu_j \quad \forall j=1,\ldots,p \tag{6}$$

In relations 4-6, vectors of $\mathbf{c}, \mathbf{a_1}, ..., \mathbf{a_m}, \mathbf{e_1}, ..., \mathbf{e_p} \in \Re^n$ and scalars of $b_1', ..., b_m', d_1', ..., d_p' \in \Re$ are the problem parameters. $\mathbf{x} \in \Re^n$ vector shows the variable of decision-making. Scalars of $\lambda_1, ..., \lambda_m, \mu_1, ..., \mu_p \in \Re$ show the dual variables. The equality constraints in relation 5 can represent the power balance constraints and demand distribution equations. The inequality constraints in relation 6 can be the production power range of the power plant units and their maximum decreasing and increasing slope, as well as the acceptable range for the power passing through the transmission lines. The dual variable related to a constraints (which in practice shows the sensitivity of the objective function to that constraint) can be divided into a set of decomposed dual covariables (DDV) according to relations (7) and (8). Scalar of $c_1, ..., c_n \in \Re$ and variables of $x_1, ..., x_n \in \Re$ are c and x vectors, respectively [19].

$$\lambda_i = \frac{\partial\left(\sum_k (c_k x_k)\right)}{\partial b_i'} = \sum_k \frac{\partial (c_k x_k)}{\partial b_i'} = \sum_k \tilde{\lambda}_i^k \quad \forall i = 1, ..., m \tag{7}$$

$$\mu_j = \frac{\partial\left(\sum_k (c_k x_k)\right)}{\partial d_j'} = \sum_k \frac{\partial (c_k x_k)}{\partial d_j'} = \sum_k \tilde{\mu}_j^k \quad \forall j = 1, ..., p \tag{8}$$

In comparison with λ_i (which shows the sensitivity of objective function relative to *i*-th function, $\tilde{\lambda}_i^k$ shows the variable cost sensitivity of x_k in $c_k x_k$ objective function, which is the production power of the k-th power plant unit. In another word, cost sensitivity of *k*-th power plant relative to the *i*-th constraints is expressed using $\tilde{\lambda}_i^k$. In order to calculate the DDVs, the optimality constraints for each of the power plant units are used, which are guaranteed due to the convexity. Some of the constraints are the initial problem conditions, the constant conditions obtained by setting the derivative of the Lagrange function equal to zero with respect to each of the variables of the initial problem and the strong duality theorem. With this definition, the optimality constraints for the *i*-th power plant unit will be expressed as a combination of constraints (5) and (6), with constraints (9), (10) and (11).

$$\mathbf{c} - \sum_i (\lambda_i \mathbf{a_i}) - \sum_j (\mu_j \mathbf{e_j}) = 0 \tag{9}$$

$$\boldsymbol{\mu} \geq 0 \tag{10}$$

$$\mathbf{c^T x} = \sum_i b_i' \lambda_i + \sum_j d_j' \mu_j \tag{11}$$

By placing relations 7 and 8 in 9, 10, and 11, relations 12-14 will govern DDVs:

$$c_l - \sum_i \left(a_{li} \tilde{\lambda}_i^k\right) - \sum_{j=1} \left(e_{lj} \tilde{\mu}_j^k\right) = 0 \quad \forall l, k = l \tag{12}$$

$$-\sum_i \left(a_{li} \tilde{\lambda}_i^k\right) - \sum_j \left(e_{lj} \tilde{\mu}_j^k\right) = 0 \quad \forall l, k \neq l \tag{13}$$

$$c_k x_k = \sum_i \left(\tilde{\lambda}_i^k b_i'\right) + \sum_j \left(\tilde{\mu}_j^k d_j'\right) \quad \forall k \tag{14}$$

If a dual coefficient is equal to zero, all DDVs corresponding to it is equal to zero, and also the sum of DDVs must be equal to the corresponding dual coefficient. The method of deriving DDVs for the model governing the electricity market is presented in Appendix 2 by relations (B1)-(B11).

Market settlement mechanism

In section 2, the share of each constraint in the cost of purchasing power from each of the market participants was mathematically modeled. Based on the effect of constraints on the accepted power of power plant units, the set of constraints governing the market is divided into three main categories, including the Playing Field Rule (PFR), which include the constraints governing the transmission network and power balance, and the constraints of the increasing slope of the production units; Non-Playing Field Rule (NPFR) which are power plant constraints and are divided into two categories: internal NPFR and external NPFR. Constraints such as the minimum production of a unit and the decreasing slope of its production are considered internal NPFR constraints for that unit, and restrictions are included minimum production and limitation of increasing and decreasing slope of other units including external NPFR restrictions. Share of PFR constraints ($G_{j,h}^{PFR}$), internal NPFR ($G_{j,h}^{(j),NPFR}$), and external NPFR ($G_{j,h}^{(-j),NPFR}$) from the accepted power of an unit ($G_{j,h}^{*}$), are shown in relations 15, 16, and 17. A detailed description of the variables shown in these relationships is provided in Appendix 1 and 2.

$$G_{j,h}^{PFR}=\left(\begin{array}{l}\underbrace{\sum_{n\in\Lambda_N,d\in\Lambda_D,t\in\Lambda_T}\left(\tilde{\phi}_{n,t}^{*j,h}IL_{d,n}L_{n,t}\right)}_{Power\ Balance}-\underbrace{\sum_{t\in\Lambda_T,t>1}\left(\tilde{\chi}_{j,t}^{*j,h}\overline{RU_j}\right)}_{Ramp\ Up}-\\ \underbrace{\sum_{l\in\Lambda_L,t\in\Lambda_T}\left(\left(\overline{\tilde{\beta}_{l,t}^{*j,h}}+\underline{\tilde{\beta}_{l,t}^{*j,h}}\right)\overline{P_l}\right)}_{Line\ Capacities}-\underbrace{\sum_{i\in\Lambda_I,t\in\Lambda_T}\left(\overline{\tilde{\varepsilon}_{i,t}^{*j,h}}U_{i,t}^{*}\overline{G_i}\right)}_{Binary\ Variables\&Max.Generation\ Limit}\end{array}\right)/\,Offer_{j,h} \qquad (15$$

$$G_{j,h}^{(j),NPFR}=\left(-\underbrace{\sum_{t\in\Lambda_T,t>1}\left(\tilde{\delta}_{j,t}^{*j,h}\overline{RD_j}\right)}_{\textbf{Ramp Down}}+\underbrace{\sum_{t\in\Theta}\left(\underline{\tilde{\varepsilon}_{j,t}^{*j,h}}U_{j,t}^{*}\underline{G_j}\right)}_{Binary\ variables\&Min.Generation\ Limit}\right)/\,Offer_{j,h} \qquad (16$$

$$G_{j,h}^{(-j),NPFR}=\left(-\underbrace{\sum_{i\in\Lambda_I-\{j\},t\in\Lambda_T,t>1}\left(\tilde{\chi}_{i,t}^{*j,h}\overline{RU_i}\right)}_{\textbf{Ramp Up}}-\underbrace{\sum_{i\in\Lambda_I-\{j\},t\in\Lambda_T,t>1}\left(\tilde{\delta}_{i,t}^{*j,h}\overline{RD_i}\right)}_{\textbf{Ramp Down}}+\underbrace{\sum_{i\in\Lambda_I-\{j\},t\in\Lambda_T}\left(\underline{\tilde{\varepsilon}_{i,t}^{*j,h}}U_{i,t}^{*}\underline{G_{i,t}}\right)}_{Binary\ variables\&Min.Generation\ Limit}\right)/\,Offer_{j,h} \qquad (17$$

The payments will be as follows by introducing the three components of the units' accepted power: A part of the power must be paid according to the proposed $G_{j,h}^{PAB}$ (Relation 18).

$$G_{j,h}^{PAB}\,@Offer_{j,h}=G_{j,h}^{*}-\max\left(G_{j,h}^{(j),NPFR},0\right) \qquad (18$$

Part of the power is only due to the internal limitation of the unit of ($G_{j,h}^{UL}$) (Relation 19).

$$G_{j,h}^{UL}\,@M_j=\max\left(G_{j,h}^{(j),NPFR},0\right) \qquad (19$$

Deprived power opportunity ($G_{j,h}^{OC}$)should be settled based on the proposed difference and the costs of the power plant unit (Relation 20).

$$G_{j,h}^{OC}\,@\left(Offer_{j,h}-M_j\right)=\begin{cases}\min\left(\overline{G_i}-G_{j,h}^{*},-G_{j,h}^{(-j),NPFR}\right) & if\quad G_{j,h}^{(-j),NPFR}<0\\ 0 & if\quad G_{j,h}^{(-j),NPFR}\geq 0\end{cases} \qquad (20$$

Numerical results

3-bus and 118-bus IEEE power systems have been used in order to review and evaluate the proposed method. The introduced model is implemented in GAMS software using a CPLEX solver.

First case study

Used 3-bus network has been shown in Figure 1. In this network, the line reactance and their power transmission capacity are assumed to be pu 0.01 and pu 2, respectively, and the planning horizon is assumed to be 7 hours. Table 3 shows the network demand in the determined time period. The information about the units in this study is according to Table 4.

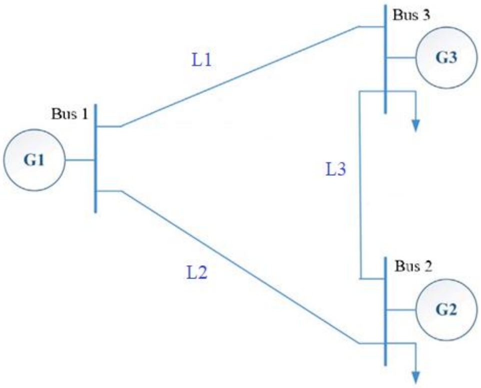


Figure 1) 3- bus network

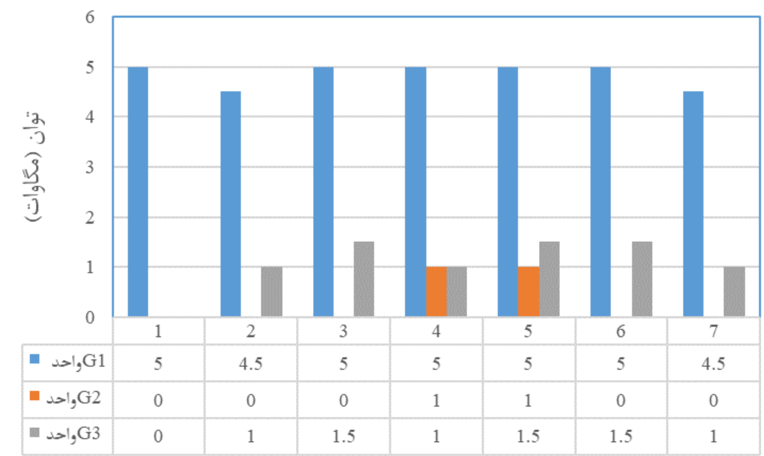
Table 3) Information of network demand in desired planning horizon

	First hour	Second hour	Third hour	Fourth hour	Fifth hour	Sixth hour	Seventh hour
2-bus	2.5	2.5	3	3.5	3.5	3	2.5
3-bus	2.5	3	3.5	3.5	4	3.5	3
Total system	5	5.5	6.5	7	7.5	6.5	5.5

Table 4) Information of power plan units in the first case study

	First generator (G1)	Second generator (G2)	Third generator (G3)
$Offer_{i,t}$	10	20	5
M_i	7	15	10
$\overline{G_i}$	5	2	2
$\underline{G_i}$	1	1	1

By implementing the market model, the results of the production arrangement in the current method used in the Iranian electricity industry and the proposed method are the same (Figure 2). Figure 3 compares the OC values of the units in both arrangements. OC results for G1 and G2 generators are the same in both modes and the only difference is observed in the G3 generator at 4 o'clock. In the current method of Iran's electricity market, OC value for G3 generator is zero at hour 4, considering the result of the feasibility check stage (impossible for G3) and the difference in the production arrangements of REQ (1 MW for G3) and REQ-pp (2 MW for G3). However, in the proposed method, considering that DDV of the minimum production power of G2 generator on the production power of G3 generator is equal to -15 ($\underline{\tilde{\varepsilon}^{G2,t4}_{G3,t4}} = -15$). The accepted power of G3 generator due to NPFR constraint of G2 generator is equal to -1 ($-\underbrace{(-1\times -15\times 1)}_{G2}/15 = -1$), and for this purpose, this generator has been subject to OC rules by 1 megawatt. Figure 4 shows UL status of the generators.



	1	2	3	4	5	6	7
واحدG1	5	4.5	5	5	5	5	4.5
واحدG2	0	0	0	1	1	0	0
واحدG3	0	1	1.5	1	1.5	1.5	1

Figure 2) Production arrangement of the 3-bus system

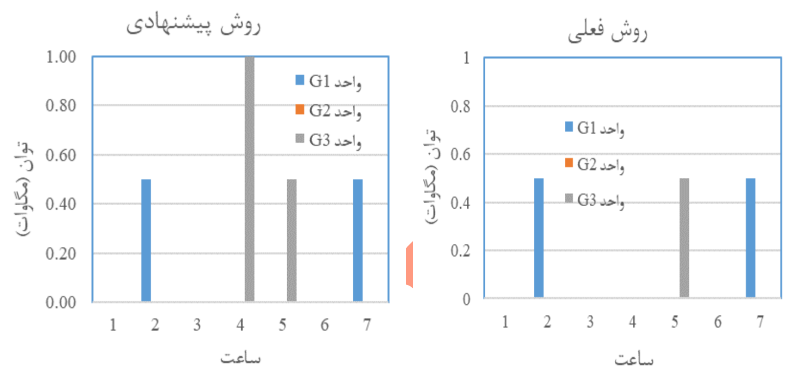


Figure 3) Comparison of power deprived in the 3-bus system

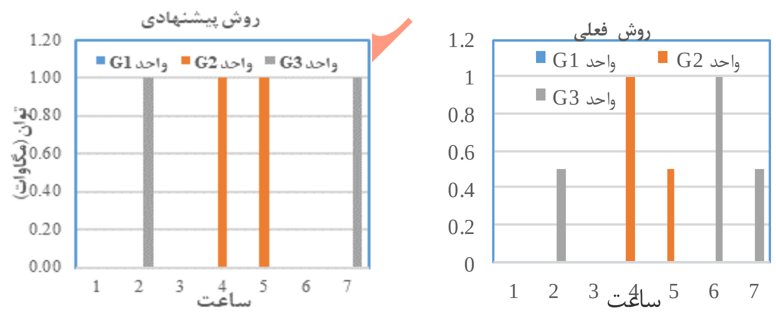


Figure 4) UL rated power comparison of generators in 3-bus system

It can be seen that the UL of the units was the same except at 6 o'clock. In the current method, the G3 unit is subject to UL regulations due to the 1 MW difference between the REQ and REQ-pp production arrangements at the 6th hour. So, this generator has not been recognized as subject to UL rules and, the results of the market settlement in this study was "Error! Reference source not found".

Table 5) Comparison between the proposed and current methods in market settlement in the 3-bus system

			T1	T2	T3	T4	T5	T6	T7	Sum
G1 Unit	Power settled based on the proposal	Current method	5	4.5	5	5	5	5	4.5	34
		suggested method	5	4.5	5	5	5	5	4.5	34

	Payment based on the proposed price	Current method	50	45	50	50	50	50	45	340
		suggested method	50	45	50	50	50	50	45	340
	Payment based on UL quantity	Current method	0	0	0	0	0	0	0	0
		suggested method	0	0	0	0	0	0	0	0
	Payment for loss of opportunity	Current method	0		0	0	0	0	**1.5**	**3**
		suggested method	0		0	0	0	0	**1.5**	**3**
	Total payment	Current method	**50**	**46.5**	50	50	50	50	**46.5**	**343**
		suggested method	**50**	**46.5**	50	50	50	50	**46.5**	**343**
G2 Unit	**Power settled based on the proposed price**	Current method	0	0	0	0	0.5	0	0	**0.5**
		suggested method	0	0	0	0	0	0	0	**0**
	Payment based on the proposed price	Current method	0	0	0	0	10	0	**0**	**10**
		suggested method	0	0	0	0	0	0	**0**	**0**
	Payment based on UL quantity	Current method	0	0	0	15	7.5	0	**0**	**22.5**
		suggested method	0	0	0	15	15	0	**0**	**30**
	Payment for loss of opportunity	Current method	0	0	0	0	0	0	0	0
		suggested method	0	0	0	0	0	0	0	0
	Total payment	Current method	0	0	0	15	17.5	0	0	32.5
		suggested method	0	0	0	15	15	0	0	30
G3 Unit	**Power settled based on the proposed price**	Current method	0	0.5	1.5	1	1.5	0.5	0.5	5.5
		suggested method	0	0	1.5	1	1.5	1.5	0	5.5
	Payment based on the proposed price	Current method	0	7.5	22.5	15	22.5	7.5	7.5	82.5
		suggested method	0	0	22.5	15	22.5	22.5	0	82.5
	Payment based on UL quantity	Current method	0	5	0	0	0	10	5	20
		suggested method	0	10	0	0	0	10	10	20
	Payment for loss of opportunity	Current method	0	0	0	0	2.5	0	0	2.5
		suggested method	0	0	0	5	2.5	0	0	7.5
	Total payment	Current method	0	12.5	22.5	15	25	17.5	12.5	105
		suggested method	0	10	22.5	20	25	22.5	10	110

Second case study

In this study, the proposed model is implemented on the IEEE 118-bus system. In this case, the planning horizon is 24 hours. Market settlement in "Error! Reference source not found" was presented and compared with the current method. The payments in the proposed method are very close to the current method and the average difference observed in market payments in the methods was almost equal to 0.03%. However, in the proposed method, UL or OC units are detected more accurately. For example, unit G12 is present in the production arrangement at 14:00, with its maximum output (0.3 pu). This means that no constraints, whether internal or external, have affected the production of this unit at this time. On the other hand, all the DDVs related to this unit were equal to zero at this hour, and accordingly, the UL power and also the OC power of this unit at 14:00 are zero. This is despite the fact that in the current method, there is a difference between the output powers in the REQ and REQ-pp arrangement for this unit. The power value in REQ is equal to 0.15 pu and the output power value of this generator in REQ-pp is equal to zero at this hour, and therefore the power value of UL is equal to 0.15 pu.

Table 6) Comparison of the current and proposed model for market settlement in the IEE 118-bus system

Time	**Payment based on sugestion**		**UL paymant**		**OC paymant**		**($\times 10^3$)**	
	Current	**Proposed**	**Proposed**	**Current**	**Proposed**	**Current**	**Proposed**	**Current**
1	631.5	620.62	12.80	0.708	0	0	0.63	0.63
2	555.7	559.72	19.18	19.31	1.4	0	0.58	0.57

3	484.4	420.54	66.1	8.02	2.52	0	0.48	0.49
4	219.9	275.57	35.82	99.64	7.5	0.74	0.31	0.32
5	351.0	374.51	31.65	62.23	0	0.76	0.40	0.41
6	495.9	498.22	17.19	20.12	2.45	0	0.51	0.51
7	603.6	623.85	2.33	19.11	0.36	0	0.62	0.62
8	712.2	705.75	11.56	9.30	0	0	0.71	0.72
9	766.0	787.89	7.4	18.8	0	0.21	0.79	0.78
10	862.6	857.62	5.16	0.291	4.54	0.01	0.86	0.86
11	882.4	853.24	21.76	0.27	0.021	0	0.87	0.88
12	802.9	783.88	23.12	8.099	2.5	0	0.80	0.81
13	731.7	721.44	26.56	17.97	20.69	0	0.76	0.74
14	630.3	668.15	18.85	51.81	0	0.58	0.68	0.68
15	871.5	819.26	25.29	0	16.99	0	0.86	0.87
16	871.8	869.82	11.6	9.68	16.76	0	0.89	0.88
17	796.6	795.22	17.15	18.4	1.06	0.05	0.81	0.81
18	855.8	859.68	17.7	17.7	6.42	0.05	0.88	0.87
19	914.5	929.52	14.31	26.73	13.51	0	0.95	0.94
20	1018.4	997.13	11.6	2.42	2.41	0	1.01	1.02
21	1053.8	946.71	58.31	2.9	0.32	0	1.00	1.05
22	823.0	832.46	36.27	60.23	5.29	0.57	0.87	0.88
23	823.5	832.82	11.41	10.64	2.6	0.08	0.84	0.83
24	770.1	781.44	0	8.02	5.33	0	0.78	0.77

Conclusion

In this article, a model based on optimization knowledge is presented in order to identify the contribution of internal and external constraints of power plant units in the production arrangement. In this model, the contribution of each of the constraints can be identified through the accepted power of the units. However, in the current method used in Iran's electricity market, the limitations of each of the units are examined in general and the effect of each of them on the accepted power of the units is not determined. The results show that in the proposed method, denied opportunity and the power that was accepted due to the limitation of the units have been recognized and calculated more accurately than the current method used in the Iranian electricity market. However, the payments in both methods are almost the same.

Appendix 1

The market exploitation problem is a MILP optimization model, which can be shown as relations (A1)-(A8) by proving the binary variables in the optimal values using solving the MILP problem. Therefore, this is a linear problem and the dual coefficients of each of the constraints are shown on their right side.

$$\min \sum_{i\in\Lambda_I, t\in\Lambda_T} \left(Offer_{i,t} G_{i,t}\right) \tag{A1}$$

$$\sum_{d\in\Lambda_D} \left(IL_{d,n} L_{d,t}\right) - \sum_{i\in\Lambda_I} \left(IG_{i,n} G_{i,t}\right) + \sum_{l\in\Lambda_L} \left(IB_{l,n} P_{l,t}\right) = 0 \quad \forall n,t \quad : \phi_{n,t} \tag{A2}$$

$$P_{l,t} - \sum_{n\in\Lambda_N} \left(IB_{l,n} \times \frac{\theta_{n,t}}{x_l} \right) = 0 \quad \forall l,t \quad : \alpha_{l,t} \tag{A3}$$

$$-\overline{P_l} \le P_{l,t} \le \overline{P_l} \quad \forall l,t \quad : \underline{\beta_{l,t}}, \overline{\beta_{l,t}} \tag{A4}$$

$$G_{i,t} - G_{i,(t-1)} \le \overline{RU_i} \quad \forall i, t>1 \quad : \chi_{i,t} \tag{A5}$$

$$G_{i,(t-1)} - G_{i,t} \le \overline{RD_i} \quad \forall i, t>1 \quad : \delta_{i,t} \tag{A6}$$

$$\underline{G_i} U_{i,t} \le G_{i,t} \le \overline{G_i} U_{i,t} \quad : \underline{\varepsilon_{i,t}}, \overline{\varepsilon_{i,t}} \quad \forall i,t \tag{A7}$$

$$\theta_{n,t} = 0 \quad : \sigma_t \quad \forall t, n=1 \tag{A8}$$

$$U_{i,t} = U_{i,t}^{*} \quad : \omega_{i,t} \tag{A9}$$

According to (A1), the operating costs of a power plant unit are equal to the product of the proposal by that unit ($Offer_{i,t}$) and its accepted power in the daily market) $G_{i,t}$ (. Equation (A2) expresses the power balance in each node of the network at each hour of the day and night. In this regard, $IL_{d,n}$ and $IG_{i,n}$ are indicators to specify the connection of the *d* load of unit *i* to node *n*. Index $IB_{l,n}$ shows the connection of line *l* to node *n* and the power passing of $P_{l,t}$ through line *l* at time *t*. Equation (A3) expresses the power distribution in line *l* with x_l reactance, in which $\theta_{n,t}$ is the angle of the its connected nodes. Equation (A4) shows the limitation of the power passing through the lines. Relations (A5) and (A6) show the constraint of the decreasing and increasing slope of production, respectively. In these equations, $\overline{RU_i}$ and $\overline{RD_i}$ are the maximum increasing and decreasing slope of production in the *i*-th unit are, respectively. in equation (A6), the power of units is restricted between the minimum value of $\underline{G_i}$ and the maximum value of $\overline{G_i}$. Equation (A8) shows that the angle of the reference node is zero. The state of placing the power plant units in the circuit is also determined by the equation (A9). It should be noted that the parameter of $U^*_{i,t}$ indicates the off and on status of the *i*-th unit, which is obtained through solving the initial MILP problem.

Appendix 2

RELATIONS (B1)-(B11) are used to calculate DDVs after solving the primary-dual problem and obtaining the dual coefficients, by establishing the optimality conditions for the *i*-th unit in the market settlement problem:

$$Offer_{i,t} - \sum_{n\in\Lambda_N}\left(G_{i,n}\tilde{\phi}^{j,h}_{n,t}\right) + \tilde{\chi}^{j,h}_{i,t} - \tilde{\chi}^{j,h}_{i,(t+1)} - \tilde{\delta}^{j,h}_{i,t} + \tilde{\delta}^{j,h}_{i,(t+1)} + \overline{\tilde{\varepsilon}^{j,h}_{i,t}} - \underline{\tilde{\varepsilon}^{j,h}_{i,t}} = 0 \quad \forall i, 1<t<24, j=i, h=t \tag{B1}$$

$$Offer_{i,t} - \sum_{n\in\Lambda_N}\left(IG_{i,n}\tilde{\phi}^{j,h}_{n,t}\right) + \tilde{\chi}^{j,h}_{i,t} - \tilde{\delta}^{j,h}_{i,t} + \overline{\tilde{\varepsilon}^{j,h}_{i,t}} - \underline{\tilde{\varepsilon}^{j,h}_{i,t}} = 0 \quad \forall i, t=24, j=i, h=t \tag{B2}$$

$$Offer_{i,t} - \sum_{n\in\Lambda_N}\left(G_{i,n}\tilde{\phi}^{j,h}_{n,t}\right) - \tilde{\chi}^{j,h}_{i,(t+1)} + \tilde{\delta}^{j,h}_{i,(t+1)} + \overline{\tilde{\varepsilon}^{j,h}_{i,t}} - \underline{\tilde{\varepsilon}^{j,h}_{i,t}} = 0 \quad \forall i, t=1, j=i, h=t \tag{B3}$$

$$-\sum_{n\in\Lambda_N}\left(IG_{i,n}\tilde{\phi}^{j,h}_{n,t}\right) + \tilde{\chi}^{j,h}_{i,t} - \tilde{\chi}^{j,h}_{i,(t+1)} - \tilde{\delta}^{j,h}_{i,t} + \tilde{\delta}^{j,h}_{i,(t+1)} + \overline{\tilde{\varepsilon}^{j,h}_{i,t}} - \underline{\tilde{\varepsilon}^{j,h}_{i,t}} = 0 \quad \forall i, 1<t<24, \left(j \neq i \, or \, h \neq t\right) \tag{B4}$$

$$-\sum_{n\in\Lambda_N}\left(IG_{i,n}\tilde{\phi}^{j,h}_{n,t}\right) + \tilde{\chi}^{j,h}_{i,t} - \tilde{\delta}^{j,h}_{i,t} + \overline{\tilde{\varepsilon}^{j,h}_{i,t}} - \underline{\tilde{\varepsilon}^{j,h}_{i,t}} = 0 \; \forall i, t=24, \left(j \neq i \, or \, h \neq t\right) \tag{B5}$$

$$-\sum_{n\in\Lambda_N}\left(IG_{i,n}\tilde{\phi}^{j,h}_{n,t}\right) - \tilde{\chi}^{j,h}_{i,(t+1)} + \tilde{\delta}^{j,h}_{i,(t+1)} + \overline{\tilde{\varepsilon}^{j,h}_{i,t}} - \underline{\tilde{\varepsilon}^{j,h}_{i,t}} = 0 \quad \forall i, t=1, \left(j \neq i \, or \, h \neq t\right) \tag{B6}$$

$$\sum_{n\in\Lambda_N}\left(IB_{l,n}\tilde{\phi}^{j,h}_{n,t}\right) + \tilde{\alpha}^{j,h}_{l,t} + \overline{\tilde{\beta}^{j,h}_{l,t}} - \underline{\tilde{\beta}^{j,h}_{l,t}} = 0 \quad \forall l, t, j, h \tag{B7}$$

$$-\sum_{l\in\Lambda_L}\left(\frac{IB_{l,n}\tilde{\alpha}^{j,h}_{l,t}}{x_l}\right) + \tilde{\sigma}^{j,h}_t = 0 \quad \forall n=1, t, j, h \tag{B8}$$

$$-\sum_{l\in\Lambda_L}\left(\frac{IB_{l,n}\tilde{\alpha}^{j,h}_{l,t}}{x_l}\right) = 0 \quad \forall n \neq 1, t, j, h \tag{B9}$$

$$-\overline{G_i}\,\overline{\tilde{\varepsilon}^{j,h}_{i,t}} + \underline{G_i}\,\underline{\tilde{\varepsilon}^{j,h}_{i,t}} + \tilde{\omega}^{j,h}_{i,t} = 0 \quad \forall i, j, h \tag{B10}$$

$$Offer_{j,h}G_{j,h} = \sum_{n\in\Lambda_N, d\in\Lambda_D, t\in\Lambda_T}\left(\tilde{\phi}^{j,h}_{n,t} IL_{d,i} L_{d,t}\right) - \sum_{l\in\Lambda_L, t\in\Lambda_T}\left(\left(\overline{\tilde{\beta}^{j,h}_{l,t}} + \underline{\tilde{\beta}^{j,h}_{l,t}}\right)\overline{P_l}\right) - \sum_{j\in\Lambda_I, h\in\Lambda_T, t>1}\left(\tilde{\chi}^{j,h}_{i,t}\overline{RU_i}\right) - \sum_{i\in\Lambda_I, t\in\Lambda_T, t>1}\left(\tilde{\delta}^{j,h}_{i,t}\overline{RD_i}\right) - \sum_{i\in\Lambda_I, t\in\Lambda_T}\left(\tilde{\omega}^{j,h}_{i,t}U^*_{i,t}\right) \quad \forall j, h \tag{B11}$$

If the dual coefficient becomes zero, all the corresponding DDVs must become zero (for example: $\tilde{\phi}^{j,h}_{n,t} = 0 \;\; if \, \phi_{n,t} = 0 \;\; \forall j, h, n, t$). On the other hand, the sum of DDVs must be equal to the corresponding double factor (for example: $\sum_{j\in\Lambda_I, h\in\Lambda_T} \tilde{\phi}^{j,h}_{n,t} = \phi_{n,t} \;\; \forall n, t$).